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Generalized Canonical Ward Identities

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Abstract In the framework of Faddeev-Senjanovic (FS) path-integral quantization, CP^1 nonlinear σ model coupled to Non-Abelian Chern-Simons (CS) fields is quantized. Generalized canonical Ward identities (WI) are deduced from the invariance of the canonical effective action under gauge transformations, which are obtained from the generators of gauge transformations, including all first-class constraints, in Dirac's sense. The generalized canonical WI has brief form and is equivalent to canonical WI under gauge transformations in Dirac's sense.

Keywords Quantum symmetries · Ward identities · CS theory · Path-integral quantization formalism

1 Introduction

CP^1 and $O(3)$ nonlinear σ models have interested some physicists [1–4] for a long time. Especially, their quantum symmetrical characteristics, the fractional spin and fractional statistics, always have attracted some people, working at quantum field and nuclear physics, to discuss [4–14]. In these papers, Noether theorem (NT) and WI were usually and widely used. For example, canonical NT had been used to discuss the fractional spin and fractional statistics of CP^1 and $O(3)$ nonlinear σ models with Abelian CS fields [4, 8, 12], or with Non-Abelian CS fields [13, 14], or with Maxwell CS term [10, 11], and it has been used to discuss the relations between fractional spins and Non-Abelian CS fields [14], too.

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WI has passed a long time for development, too. Since WI relating the Green functional in QED were first obtained by Ward [15] and Takahashi [16], it had been generalized by Slavnov [17] and Taylor [18], and it had been introduced into constrained canonical Hamiltonian systems by Li [19–21]. They are very useful and effective tools in modern quantum fields and particles. In past years, WI and generalizations have been generalized to supersymmetry [22], superstring [23] theories and other fields. All most of these WI and generalizations are deduced from the invariance of a canonical action under gauge transformations, not a canonical effective action. In this paper, a generalized WI will be deduced from the invariance of a canonical effective action under Diarc's gauge transformations, which are applied to CP^1 nonlinear sigma model coupling with Non-Abelian Chern-Simons fields to give results in brief form.

Dirac's conjecture is well known to everybody, that is, all the first-class constraints in a constrained Hamiltonian system are generators of gauge transformations. Generators of gauge transformation can give gauge transformation, under which some actions are invariant which will bring out unchanged variables or some invariant identities [26]. For a constrained Hamiltonian system, a canonical effective Lagrangian is more general and exact than a canonical Lagrangian, because it contains a canonical Lagrangian, all the constraints including first-class and second-class, and the gauge conditions corresponding to the firsclass constraints, and ghost fields. In Dirac's sense, the gauge conditions are introduced to change all first-class constraints into second-class constraints, but all second-class constraints are invariant under gauge transformations [27–29]. It is, therefore, very easy to see, once ghost fields are invariant under the gauge transformations, that the canonical effective action would be invariant, from which generalized canonical WI are given.

Our paper is recognized as follows. In Sect. 2, CP^1 nonlinear sigma model coupled to Non-Abelian Chern-Simons fields is considered as an example to quantized in the framework of Faddeev-Senjanovic path-integral quantization. In Sect. 3, the generalized WI are deduced from the invariance of the canonical effective action under the Dirac's gauge transformation. In Sect. 4, we conclude that the generalized WI is equivalent to canonical WI under the gauge transformations in Dirac's sense, but it has brief form.

2 Quantization

We consider CP^1 nonlinear σ model coupled to Non-Abelian CS fields in $(2+1)$ dimensional space-time, the Lagrangian density is [30, 31]

$$\mathcal{L} = \frac{1}{f}(D_\mu Z_k)^*(D_\mu Z_k) + \frac{n}{4\pi}\varepsilon^{\mu\nu\lambda}\left(\partial_\mu A_\nu^a A_\lambda^a + \frac{1}{3} f_{bc}^a A_\mu^a A_\nu^b A_\lambda^c\right) \tag{1}$$

where f is coupling constant, subindexes $k = 1, 2$, $\mu, \nu, \lambda = 0, 1, 2$ and Z_k is a two-component complex fields which satisfies the following constraint

$$Z_k Z_k^* = |Z_1|^2 + |Z_2|^2 = 1, \tag{2}$$

and the covariant derivative $D_\mu = \partial_\mu - iT^a A_\mu^a$, A_μ are the Non-Abelian CS fields, and T^a are generators of gauge group, $\{T^a, T^b\} = if_{bc}^a T^c$, $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$. The gauge invariance of Non-Abelian CS fields requires the parameter $n = 0, \pm 1, \pm 2, \pm 3, \ldots$. The canonical momenta π_k, π_k^* with respect to Z_k, Z_k^*, and π_i^a, π_0^a with respect to A_i, A_0 are

$$\pi_k = \frac{\partial \mathcal{L}}{\partial \dot{Z}_k} = \frac{1}{f}(D_0 Z_k)^*, \tag{3}$$

$$\pi_k^* = \frac{\partial \mathcal{L}}{\partial \dot{Z}_k^*} = \frac{1}{f} D_0 Z^k, \tag{4}$$

$$\pi_i^a = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = \frac{n}{8\pi} \varepsilon^{ij} A_j^a, \tag{5}$$

$$\pi_0^a = \frac{\partial \mathcal{L}}{\partial \dot{A}_0} = 0, \tag{6}$$

respectively, and the convention $\varepsilon^{012} = \varepsilon^{12} = 1$ is used. Equations (5) and (6) both are primary constraints. According to all of the previous canonical momenta, the canonical Hamiltonian of the considered system can be written as

$$\begin{aligned} H_c &= \int d^2x \, \mathcal{H}_c \\ &= \int d^2x \left[f \pi_k \pi_k^* + \frac{1}{f} (D_i Z_k)^* (D_i Z_k) - A_0^a \left(\frac{n}{8\pi} \varepsilon^{ij} F_{ij}^a + J_0^a \right) \right] \end{aligned} \tag{7}$$

where

$$F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g f_{bc}^a A_i^b A_j^c, \tag{8}$$

$$J_0^a = -i (\pi_k Z_k - Z_k^* \pi_k^*). \tag{9}$$

Furthermore, the corresponding total Hamiltonian can be given by

$$H_T = \int d^2x \left[\mathcal{H}_c + \lambda^a \pi_0^a + \mu_i^a \left(\pi_i^a - \frac{n}{8\pi} \varepsilon_{ij} A^{ja} \right) + \mu_0 (Z_k Z_k^* - 1) \right]. \tag{10}$$

Following Dirac-Bergmann method, only the consistency conditions of the primary constraints, $\{\pi_0^a, H_T\} \approx 0$, and $\{Z_k Z_k^* - 1, H_T\} \approx 0$, lead to two secondary constraints as

$$\frac{n}{8\pi} \varepsilon^{ij} F_{ij}^a + J_0^a \approx 0, \tag{11}$$

$$\pi^k Z_k + (\pi^k Z_k)^* \approx 0. \tag{12}$$

The stationarity of the rest primary constraints lead to the equation for determining the Lagrangian multipliers λ_i^a. The consistency conditions of the secondary constraints (11) and (12) do not give rise to more additional constraints. Similar to the Non-Abelian CS theories [32], we choose the linear combination of constraints as

$$\Lambda_0^a = \pi_0^a \approx 0, \tag{13}$$

$$\Lambda_1^a = (D_i \pi^i)^a + \frac{n}{8\pi} \varepsilon^{ij} \partial_i A_j^a + J_0^a \approx 0, \tag{14}$$

$$\theta_1 = \pi_1^a - \frac{n}{8\pi} A^{2a} \approx 0, \tag{15}$$

$$\theta_2 = \pi_2^a + \frac{n}{8\pi} A^{1a} \approx 0, \tag{16}$$

$$\theta_3 = Z_k Z_k^* - 1 \approx 0, \tag{17}$$

$$\theta_4 = \pi_k Z_k^* + \pi_k^* Z_k \approx 0. \tag{18}$$

It is easy to check that only Λ_0^a and Λ_1^a both are first-class constraints, and the rest θ_1, θ_2, θ_3 and θ_4 are second-class constraints.

In the framework of FS path-integral quantization, gauge conditions should be introduced to correspond to first-class constraint each other. We consider the Coulomb gauge

$$\Omega_2^a = \partial^i A_i^a \approx 0 \tag{19}$$

and its consistency condition, $\dot{\Omega}_2^a \approx 0$, as another gauge condition Ω_1^a as

$$\Omega_1^a = \{\Omega_2^a, H_T\} = \partial^i \pi_i^a + \nabla^2 A_0^a - f_{bc}^a A_i^b \partial^i A_0^c \approx 0 \tag{20}$$

with respect to first-class constraints (13) and (14), respectively. Introduced exterior sources $J = (J_\mu^a, J_k, J_k^*)$ with respect to the field variables $\phi = (A_\mu^a, Z_k, Z_k^*)$, and exterior sources $K = (K_a^\mu, K^k, K^{k*})$ correspond to the field variables $\pi = (\pi_a^\mu, \pi^k, \pi^{k*})$, the phase-space generating functional of Green function can be written as

$$\begin{aligned} Z[J] = \int & \mathcal{D}A_\mu^a \mathcal{D}\pi_a^\mu \mathcal{D}Z_k \mathcal{D}\pi^k \mathcal{D}Z_k^* \mathcal{D}\pi^{k*} \delta(\Lambda)\delta(\Omega)\delta(\theta) \det|\Lambda_k^a, \Omega^k| [\det|\theta_i^a, \theta_j^b|]^{\frac{1}{2}} \\ & \times \exp\left\{ i \int d^4x (\mathcal{L}^P + J\phi + K\pi) \right\}. \end{aligned} \tag{21}$$

In above equation (21), it is easy to calculate that

$$\det|\Lambda_k^a, \Omega^k| = \det|M^{ab}|\delta^{(2)}(\vec{x} - \vec{y})| \tag{22}$$

where

$$M^{ab} = \begin{bmatrix} -\delta^{ab}\nabla^2 + g f_{bc}^a A_i^c \partial^i & 0 \\ 0 & -\delta^{ab}\nabla^2 + g f_{bc}^a A_i^c \partial^i \end{bmatrix} = -\delta^{ab}\partial^i\partial_i + g f_{bc}^a A_i^c \partial^i. \tag{23}$$

The factor $\det|\{\Lambda_k^a, \Omega^k\}|\delta(\partial^i A_i^a)$ in the right side of expression (21) can be replaced by $\det|M_L \delta(\partial^\mu A_\mu^a)|$ [33], where

$$M_L = (\delta^{ab}\partial^\mu\partial_\mu - g f_{bc}^a A_i^c \partial^\mu)\delta^{(2)}(\vec{x} - \vec{y}). \tag{24}$$

Using the integral properties of the Grassmann variables $C(x)$ and $\bar{C}(x)$, we obtain

$$\det|\{A_a(x), B^b(y)\}| = \int \mathcal{D}\bar{C}_a \mathcal{D}C_b \exp\left[i \int d^3y \bar{C}_a(x)\{A_a(x), B^b(y)\}C_b(y) \right]. \tag{25}$$

According to the properties of δ function and the integral characters of Grassmann variables $C(x)$ and $\bar{C}(x)$, the generating functional (21) can be rewritten as

$$\begin{aligned} Z[J, K, \bar{\xi}, \xi, X, Y] = \int & \mathcal{D}A_\mu^a \mathcal{D}\pi_a^\mu \mathcal{D}Z_k \mathcal{D}\pi^k \mathcal{D}Z_k^* \mathcal{D}\pi^{k*} \mathcal{D}\bar{C}^a \mathcal{D}C^a \\ & \times \exp\left\{ i \int d^4x (\mathcal{L}_{eff}^P + J\phi + K\pi + \bar{\xi}_a C^a \right. \\ & \left. + \bar{C}^a \xi_a + X_k^a \lambda_a^k + Y_l \mu^l) \right\} \end{aligned} \tag{26}$$

where

$$\mathcal{L}_{eff}^{P}=\mathcal{L}^{P}-\partial_{\mu}\bar{C}^{a}D_{\nu b}^{a}C^{b}+\lambda_{a}^{k}\Lambda_{k}^{a}+\mu^{l}\theta_{l}-\frac{1}{2\alpha_{1}}(\Omega_{1}^{a})^{2}-\frac{1}{2\alpha_{2}}(\partial^{\mu}A_{\mu}^{a})^{2} \tag{27}$$

is called canonical effective Lagrangian density, in which λ_a^k and μ^l are multiplier fields, $\bar{\xi}_a$ and ξ_a are exterior sources introduced with respect to ghost $\bar{C}^a$ and C^a, respectively, and X_k^a and Y_l are exterior sources introduced with respect to multiplier fields λ_a^k and μ^l, respectively.

3 Generalized Canonical WI

In order to discuss the quantum symmetries, generalized canonical WI, we construct gauge transformations for the system (1) first. Assuming Dirac's conjecture valid, all the first-class constraints are the generators of the gauge transformations, and basing on the two first-class constraints (13) and (14), the generator of the gauge transformations can be expressed as

$$\begin{aligned}G&=\int d^{2}x[\dot{\varepsilon}(x)\Lambda_{0}^{a}+\varepsilon(x)\Lambda_{1}^{a}]\\&=\int d^{2}x\left[\dot{\varepsilon}\pi_{0}^{a}+\varepsilon\left(D^{i}\pi_{i}^{a}+\frac{n}{8\pi}\varepsilon^{ij}\partial_{i}A_{j}^{a}+J_{0}^{a}\right)\right].\end{aligned} \tag{28}$$

This generator can generate the gauge transformations required as

$$\begin{cases}\delta Z_{k}=\{Z_{k},G\}=-i\varepsilon Z_{k},\\ \delta Z_{k}^{*}=\{Z_{k}^{*},G\}=i\varepsilon Z_{k}^{*},\\ \delta A_{0}^{a}=\{A_{0}^{a},G\}=\dot{\varepsilon},\\ \delta A_{i}^{a}=\{A_{i}^{a},G\}=\varepsilon D_{i}\delta(\vec{x}-\vec{y}),\\ \delta\pi^{k}=\{\pi^{k},G\}=i\varepsilon\pi_{k},\\ \delta\pi^{k*}=\{\pi^{k*},G\}=-i\varepsilon\pi_{k}^{*},\\ \delta\pi_{a}^{i}=\{\pi_{a}^{i},G\}=-\dfrac{n}{8\pi}\varepsilon\varepsilon^{ij}\partial_{j}\delta(\vec{x}-\vec{y}),\end{cases} \tag{29}$$

and the gauge transformations of all the rest variables are zeros. Furthermore, we will have studied, under the gauge transformations (29), the variation of the canonical effective action is given by

$$\begin{aligned}\Delta I_{eff}^{P}=&\int d^{3}x\varepsilon\left[\left(-\dot{\pi}^{k}-\frac{\delta H_{eff}}{\delta Z_{k}}\right)\delta Z_{k}+\left(-\dot{\pi}^{k*}-\frac{\delta H_{eff}}{\delta Z_{k}^{*}}\right)\delta Z_{k}^{*}+\left(-\dot{\pi}_{a}^{i}-\frac{\delta H_{eff}}{\delta A_{i}^{a}}\right)\delta A_{i}^{a}\right]\\&+\int d^{3}x\left(-\dot{\pi}_{a}^{0}-\frac{\delta H_{eff}}{\delta A_{0}^{a}}\right)\delta A_{0}^{a}\\&+\int d^{3}xD[\pi^{k}\delta Z_{k}+\pi^{k*}\delta Z_{k}^{*}+\pi_{a}^{i}\delta A_{i}^{a}+\pi_{a}^{0}\delta A_{0}^{a}]\end{aligned} \tag{30}$$

where $H_{eff}=\int d^2x\mathcal{H}_{eff}$ is an canonical effective Hamiltonian with respect to the canonical effective Lagrangian $L_{eff}^P=\int_V d^2x\mathcal{L}_{eff}^P$. It is supposed that the Jacobian of the gauge transformations (29) equals to unity, and the generating functional (26) is invariant under the gauge transformations (29), we can obtain the following form

$$Z[J,K,\bar{\xi},\xi,X,Y]=\int \mathcal{D}\phi\mathcal{D}\pi\mathcal{D}\bar{C}^a\mathcal{D}C^a\left[1+i\Delta I_{eff}^P+i\varepsilon\int d^3x(J\delta\phi+K\delta\pi)\right]_{\substack{\pi\to\partial\mathcal{L}/\partial\dot{\phi}\\ \phi\to -i\delta/\delta J}}$$
$$\times Z[J,K,\bar{\xi},\xi,X,Y]. \tag{31}$$

According to the previous equations (30) and (31), the phase-space generating functional (26) satisfies the following form

$$\int d^3x\left\{\varepsilon\left[\left(-\dot{\pi}^k-\frac{\delta H_{eff}}{\delta Z_k}\right)\delta Z_k+\left(-\dot{\pi}^{k*}-\frac{\delta H_{eff}}{\delta Z_k^*}\right)\delta Z_k^*+\left(-\dot{\pi}_a^i-\frac{\delta H_{eff}}{\delta A_i^a}\right)\delta A_i^a\right]\right.$$
$$+\left(-\dot{\pi}_a^0-\frac{\delta H_{eff}}{\delta A_0^a}\right)\delta A_0^a+D[\pi^k\delta Z_k+\pi^{k*}\delta Z_k^*+\pi_a^i\delta A_i^a+\pi_a^0\delta A_0^a]$$
$$\left.+(J\delta\phi+K\delta\pi)\right\}_{\substack{\pi\to\partial\mathcal{L}/\partial\dot{\phi}\\ \phi\to -i\delta/\delta J}}Z[J,K,\bar{\xi},\xi,X,Y]=0. \tag{32}$$

Under the gauge transformations (29), the canonical action I^P is invariant that is well known. And it is well known that under gauge transformations (29) all the second-class constraints are invariant, for which the term $I_m=\int\mathcal{L}_m d^2x$ is also invariant because that all the first-class constraints have been changed into second-class ones by introducing the corresponding gauge conditions, and the ghost term I_g, $I_g=\int\mathcal{L}_g d^2x=\int(\partial_\mu\bar{C}^a D_{\nu b}^a C^b)d^2x$, is also invariant because that $\delta C(x)=0$ and $\delta\bar{C}(x)=0$ in (29). To summery all previous analysis, it is easy to know that the canonical effective action I_{eff}^P is invariant [34, 35]. Therefore, (32) can be simplified as

$$\int d^3x\{(J\delta\phi+K\delta\pi)\}_{\pi\to\partial\mathcal{L}/\partial\dot{\phi},\phi\to -i\delta/\delta J}Z[J,K,\bar{\xi},\xi,X,Y]=0. \tag{33}$$

Substituting the gauge transformations (29) into (33), we can obtain

$$\int d^3x\left\{\varepsilon\left[J^k\left(-\frac{\delta}{\delta J^k}\right)+J^{k*}\left(\frac{\delta}{\delta J^{k*}}\right)-\partial_0 J_a^0+J_a^i((D_i)^a)+K_k\left(\frac{\delta}{\delta K_k}\right)\right.\right.$$
$$\left.\left.+K_k^*\left(-\frac{\delta}{\delta K_k^*}\right)+K_i^a\left(-\frac{n}{8\pi}\varepsilon^{ij}(\partial_j)^a\right)\right]+D(\varepsilon J_a^0)\right\}_{\substack{\pi\to\partial\mathcal{L}/\partial\dot{\phi}\\ \phi\to -i\delta/\delta J}}$$
$$\times Z[J,K,\bar{\xi},\xi,X,Y]=0 \tag{34}$$

or

$$\left[-J^k\frac{\delta}{\delta J^k}+J^{k*}\frac{\delta}{\delta J^{k*}}-\partial_0 J_a^0-D_i^a J_a^i+K_k\frac{\delta}{\delta K_k}\right.$$
$$\left.-K_k^*\frac{\delta}{\delta K_k^*}+\frac{n}{8\pi}\varepsilon^{ij}\partial_i^a K_j^a\right]Z[J,K,\bar{\xi},\xi,X,Y]=0. \tag{35}$$

Let $Z[J,K,\bar{\xi},\xi,X,Y]=\exp\{iW[J,K,\bar{\xi},\xi,X,Y]\}$ and use the definition of the generating functional of proper vertices $\Gamma[\phi,\pi,C^a,\bar{C}^a,\lambda,\mu]$ which is given by performing a functional Legendre transformation on $W[J,K,\bar{\xi},\xi,X,Y]$,

$$\Gamma[\phi,\pi,C^a,\bar{C}^a,\lambda,\mu]=W[J,K,\bar{\xi},\xi,X,Y] - \int d^2x(J\phi+K\pi+\bar{\xi}_a C^a+\xi_a\bar{C}^a+X_k^a\lambda_a^k+Y_l\mu^l) \quad (36)$$

and

$$\begin{aligned}
\frac{\delta W}{\delta J}&=\phi \quad\Longleftrightarrow\quad \frac{\delta\Gamma}{\delta\phi}=-J,\\
\frac{\delta W}{\delta K}&=\pi \quad\Longleftrightarrow\quad \frac{\delta\Gamma}{\delta\pi}=-K,\\
\frac{\delta W}{\delta\bar{\xi}_a}&=C^a \quad\Longleftrightarrow\quad \frac{\delta\Gamma}{\delta C^a}=-\bar{\xi}_a,\\
\frac{\delta W}{\delta\xi_a}&=\bar{C}^a \quad\Longleftrightarrow\quad \frac{\delta\Gamma}{\delta\bar{C}^a}=-\xi_a,\\
\frac{\delta W}{\delta X_k^a}&=\lambda_a^k \quad\Longleftrightarrow\quad \frac{\delta\Gamma}{\delta\lambda_a^k}=-X_k^a,\\
\frac{\delta W}{\delta Y_l}&=\mu^l \quad\Longleftrightarrow\quad \frac{\delta\Gamma}{\delta\mu^l}=-Y_l.
\end{aligned} \quad (37)$$

Then (35) can be simplified as

$$Z_k\frac{\delta\Gamma}{\delta Z_k}-Z_k^*\frac{\delta\Gamma}{\delta Z_k^*}+\partial_0\frac{\delta\Gamma}{\delta A_0^a}+D_i^a\frac{\delta\Gamma}{\delta A_i^a}-\pi^k\frac{\delta\Gamma}{\delta\pi^k}+\pi^{k*}\frac{\delta\Gamma}{\delta\pi^{k*}}-\frac{n}{8\pi}\varepsilon^{ij}\partial_i^a\frac{\delta\Gamma}{\pi_j^a}=0. \quad (38)$$

By functionally differentiating (38) with respect to Z_k and Z_k^*, and set all fields equal to zero, $\phi=\pi=\bar{C}^a=C^a=\lambda_a^k=\mu^l=0$, we can get a peculiar generalized canonical WI as

$$\partial_{x_1}^\mu\frac{\delta^3\Gamma}{\delta Z_k^*\delta Z_k\delta A_\mu^a}=\delta(x_1-x_2)\frac{\delta^2\Gamma}{\delta Z_k\delta Z_k^*}-\delta(x_1-x_3)\frac{\delta^2\Gamma}{\delta Z_k^*\delta Z_k} + \frac{n}{8\pi}\varepsilon^{ij}\partial_i^a\frac{\delta^3\Gamma}{\delta Z_k^*\delta Z_k\delta\pi_j^a}. \quad (39)$$

Similarly, differentiating (38) many times with respect to field variables and setting all fields equal to zero, one can obtain various generalized canonical WI for proper vertices.

If we consider that under the transformations (29) the canonical action $I^P=\int d^3x\mathcal{L}^P$ is invariant, the change of canonical effective action will be rewritten as

$$\Delta I_{eff}^P=\int d^3x\varepsilon\left[-\frac{\delta H'_{eff}}{\delta Z_k}\delta Z_k-\frac{\delta H'_{eff}}{\delta Z_k^*}\delta Z_k^*-\frac{\delta H'_{eff}}{\delta A_i^a}\delta A_i^a-\frac{\delta H'_{eff}}{\delta A_0^a}\delta A_0^a\right] \quad (40)$$

where

$$H'_{eff}=\int d^3x\left[\partial_\mu\bar{C}^aD_{vb}^aC^b-\lambda_a^k\Lambda_k^a-\mu^l\theta_l+\frac{\alpha_1}{2}(\Omega_1^a)^2+\frac{\alpha_2}{2}(\partial^\mu A_\mu^a)^2\right]. \quad (41)$$

Therefore, (33) have to be replaced by the following form

$$\int d^3x \left\{ \left[-\frac{\delta H'_{\mathit{eff}}}{\delta Z_k}\delta Z_k - \frac{\delta H'_{\mathit{eff}}}{\delta Z_k^*}\delta Z_k^* - \frac{\delta H'_{\mathit{eff}}}{\delta A_i^a}\delta A_i^a - \frac{\delta H'_{\mathit{eff}}}{\delta A_0^a}\delta A_0^a \right] + (J\delta\phi + K\delta\pi) \right\}_{\substack{\pi\to\partial\mathcal{L}/\partial\dot{\phi} \\ \phi\to -i\delta/\delta J}}$$
$$\times Z[J, K, \bar{\xi}, \xi, X, Y] = 0. \tag{42}$$

If you would like to get the resultant equation (38) from (42), you have to confirm the following equation

$$-\frac{\delta H'_{\mathit{eff}}}{\delta Z_k}\delta Z_k - \frac{\delta H'_{\mathit{eff}}}{\delta Z_k^*}\delta Z_k^* - \frac{\delta H'_{\mathit{eff}}}{\delta A_i^a}\delta A_i^a - \frac{\delta H'_{\mathit{eff}}}{\delta A_0^a}\delta A_0^a = 0. \tag{43}$$

This work is not an easy thing to be done directly. But in Dirac's sense, this expression can be demonstrated easily because under the gauge transformation (29) the variances of the ghost fields, and the variances of the second-class constraints, and the first-class ones and their corresponding gauge conditions are zeros. Therefore, under Dirac's gauge transformations the invariance of canonical effective action is equivalent to the ones of canonical action. Furthermore, (38) has special generalities in fields and particles, from which many Feymann rules can be given.

4 Conclusion

In many papers, for constrained canonical systems canonical WI had been discussed from local gauge transformations [19, 36, 37], non-local gauge transformations [38–40], and global gauge transformations [41, 42]. They were seldom to get the gauge transformations deduced from the generators of gauge transformations in Dirac's sense. In a book [43], the authors and their coworkers had discussed canonical WI from gauge transformations in Dirac's sense, but they were based on canonical action invariant under the gauge transformations, for example WI (42). But we deduced WI based on the canonical effective action invariant under the gauge transformations in Dirac's sense, for example WI (33). This method can be used widely to many practical physical systems, because two reasons, one is that all the first-class constraints can be changed into second-class constraints by introducing suitable gauge conditions, all the second-class constraints are invariant under the gauge transformations in Dirac's sense, the other is that the generators of the gauge transformations are combined by primary and secondary first-class constraints which are deduced before ghost fields introduced, in other words, the variance of all the ghost fields can be vanished from the canonical effective action. It is well known that a canonical effective Lagrangian consists of canonical Lagrangian, and all the constraints including first-class and second-class, and the chosen gauge conditions corresponding to the first-class constraints, and the ghost fields. In Dirac's sense, the rest three parts in the canonical effective action except the canonical action,the front two terms are invariant and the third can be vanished under the gauge transformation in Dirac's sense, which means that the invariance of the canonical effective action is equivalent to the invariance of the canonical action.

The most advantage of the above derivation for WI of proper vertices is that one does not need to carry out explicitly the integration over the canonical momenta in the phase-space generating functional as one usually does. In a general case it is not possible to carry out this integration. But we still need to pay our attention on that all constraints in constrained canonical systems (including all first-class constraints, second-class constraints and gauge conditions) are invariant under the gauge transformations produced by the generators in Dirac's sense. Many important constrained physical systems need to discuss further.

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